

Magnon Exchange Mechanism of Superconductivity: $ZrZn_2$, $URhGe$

Naoum Karchev*

Department of Physics, University of Sofia, 1126 Sofia, Bulgaria

The magnon exchange mechanism of superconductivity was developed to explain in a natural way the fact that the superconductivity in UGe_2 , $ZrZn_2$ and $URhGe$ is confined to the ferromagnetic phase. The order parameter is a spin antiparallel component of a spin-1 triplet with zero spin projection. The transverse spin fluctuations are pair forming and the longitudinal ones are pair breaking. In the present paper, a superconducting solution, based on the magnon exchange mechanism, is obtained which closely matches the experiments with $ZrZn_2$ and $URhGe$. The onset of superconductivity leads to the appearance of complicated Fermi surfaces in the spin up and spin down momentum distribution functions. Each of them consist of two pieces, but they are simple-connected and can be made very small by varying the microscopic parameters. As a result, it is obtained that the specific heat depends on the temperature linearly, at low temperature, and the coefficient $\gamma = \frac{C}{T}$ is smaller in the superconducting phase than in the ferromagnetic one. The absence of a quantum transition from ferromagnetism to ferromagnetic superconductivity in a weak ferromagnets $ZrZn_2$ and $URhGe$ is explained accounting for the contribution of magnon self-interaction to the spin fluctuations' parameters. It is shown that in the presence of an external magnetic field the system undergoes a first order quantum phase transition.

74.20.Mn, 75.50.Cc, 75.10.Lp

Very recently ferromagnetic superconductivity (f-superconductivity) has been observed in UGe_2 [1], $ZrZn_2$ [2] and $URhGe$ [3]. The superconductivity is confined to the ferromagnetic phase. Ferromagnetism and superconductivity are believed to arise due to the same band electrons. The persistence of ferromagnetic order within the superconducting phase has been ascertained by neutron scattering. The specific heat anomaly associated with the superconducting transition in these materials appears to be absent.

At ambient pressure UGe_2 is an itinerant ferromagnet below the Curie temperature $T_c = 52K$, with low-temperature ordered moment of $\mu_s = 1.4\mu_B/U$. With increasing pressure the system passes through two successive quantum phase transition, from ferromagnetism to f-superconductivity at $P \sim 10$ kbar, and at higher pressure $P_c \sim 16$ kbar to paramagnetism [1,4]. At the pressure where the superconducting transition temperature is a maximum $T_{sc} = 0.8K$, the ferromagnetic state is still stable with $T_c = 32K$, and an ordered moment about $1.0\mu_B/U$ [1,4,5]. The specific heat coefficient $\gamma = \frac{C}{T}$ increases steeply near 11 kbar and retains a large and nearly constant value [5].

The ferromagnets $ZrZn_2$ and $URhGe$ are superconducting at ambient pressure with superconducting critical temperatures $T_{sc} = 0.29K$ and $T_{sc} = 0.25K$ respectively. $ZrZn_2$ is ferromagnetic below the Curie temperature $T_c = 28.5K$ with low-temperature ordered moment of $\mu_s = 0.17\mu_B$ per formula unit, while for $URhGe$ $T_c = 9.5K$ and $\mu_s = 0.42\mu_B$. The low Curie temperatures and small ordered moments indicate that compounds are close to a ferromagnetic quantum critical point. A large jump in the specific heat, at the temperature where the resistivity becomes zero, is observed in $URhGe$. At low temperature the specific heat coefficient γ is twice smaller than in the ferromagnetic phase.

The most popular theory of f-superconductivity is based on the paramagnon exchange mechanism [6,7]. The order parameters are spin parallel components of the spin triplet. The superconductivity in $ZrZn_2$ was predicted, but the theory meets many difficulties. In order to explain the absence of superconductivity in paramagnetic phase it was accounted for the magnon paramagnon interaction and proved that the critical temperature is much higher in the ferromagnetic phase than in the paramagnetic one [8]. To the same purpose, the Ginzburg-Landau mean-field theory was modified with an exchange-type interaction between the magnetic moments of triplet-state Cooper pairs and the ferromagnetic magnetization density [9]. In [10] the authors make the important assumption that only majority spin fermions form pairs. Then, only minority spin fermions contribute to the asymptotic of the specific heat, and the coefficient $\gamma = \frac{C}{T}$ is twice smaller in the superconducting phase. The result closely matches the experiments with $URhGe$ [3], but does not resemble the experimental results for UGe_2 and $ZrZn_2$. The assumption seems to be doubtful for systems with very small ordered moment. Despite of the efforts, the improved theory of paramagnon induced superconductivity can not cover the whole variety of properties of f-superconductivity.

In the present paper an itinerant system is considered in which the spin- $\frac{1}{2}$ fermions $c_\sigma(\vec{x})(c_\sigma^\dagger(\vec{x}))$ responsible for the ferromagnetism are the same quasiparticles which form the Cooper pairs. The exchange of spin fluctuations leads to an effective four fermion theory. It describes the interaction of the components of spin-1 composite fields ($\uparrow\uparrow, \uparrow\downarrow + \downarrow\uparrow, \downarrow\downarrow$) which have a projection of spin 1, 0 and -1 respectively, and the interaction of the spin singlet composite fields $\uparrow\downarrow - \downarrow\uparrow$. The spin singlet fields' interaction is repulsive and does not contribute to the superconductivity [11]. The spin parallel fields' interactions are due to the exchange of paramagnons and do not contribute to the magnon-mediated superconductivity.

ity. The relevant interaction is that of the $\uparrow\downarrow + \downarrow\uparrow$ fields. The potential of this interaction has an attracting part due to exchange of magnons and a repulsive part due to exchange of paramagnons.

By means of the Hubbard-Stratanovich transformation one introduces $\uparrow\downarrow + \downarrow\uparrow$ composite field and then the fermions can be integrated out. The obtained free energy is a function of the composite field and the integral over the composite field can be performed approximately by means of the steepest descend method. To this end one sets the first derivative of the free energy with respect to composite field equal to zero, this is the gap equation, and looks for a solution which minimizes the free energy.

The gap is an antisymmetric function $\Delta(-\vec{k}) = -\Delta(\vec{k})$, so that the expansion in terms of spherical harmonics $Y_{lm}(\Omega_{\vec{k}})$ contains only terms with odd l . I assume that the component with $l = 1$ and $m = 0$ is nonzero and the other ones are zero

$$\Delta(\vec{k}) = \Delta_{10}(k) \sqrt{\frac{3}{4\pi}} \cos \theta. \quad (1)$$

Expanding the potential in terms of Legendre polynomial P_l one obtains that only the component with $l = 1$ contributes the gap equation. The potential $V_1(p, k)$ has the form,

$$V_1(p, k) = \frac{3M}{\rho} \left[\frac{p^2 + k^2}{4p^2k^2} \ln \left(\frac{p+k}{p-k} \right)^2 - \frac{1}{pk} \right] - \frac{3M}{\rho} \beta \left[\frac{p^2 + k^2}{4p^2k^2} \ln \frac{r' + (p+k)^2}{r' + (p-k)^2} - \frac{1}{pk} \right], \quad (2)$$

where M is zero temperature dimensionless magnetization of the system per lattice site and ρ is the spin stiffness constant which is proportional to M ($\rho = M\rho_0$) The constants β, ρ_0 and b are phenomenological ones subject to the relation $\beta = \frac{\rho}{2Mb} = \frac{\rho_0}{2b} > 1$, and $r' = \frac{r}{b} \ll 1$, where the parameter r is the inverse static longitudinal magnetic susceptibility, which measures the deviation from quantum critical point. A straightforward analysis shows that for a fixed p , the potential is positive when k runs an interval around p ($p - \Lambda, p + \Lambda$), where Λ is approximately independent on p . In order to allow for an explicit analytic solution, I introduce further simplifying assumptions by neglecting the dependence of $\Delta_{10}(k)$ on k ($\Delta_{10}(k) = \Delta_{10}(p_f) = \Delta, p_f = \sqrt{2\mu m}$) and setting $V_1(p_f, k)$ equal to a constant V_1 within interval $(p_f - \Lambda, p_f + \Lambda)$ and to zero elsewhere.

To ensure that the fermions which form Cooper pairs are the same as those responsible for spontaneous magnetization, one has to consider the equation for the magnetization

$$M = \frac{1}{2} \langle c_{\uparrow}^{\dagger} c_{\uparrow} - c_{\downarrow}^{\dagger} c_{\downarrow} \rangle \quad (3)$$

as well. Then the system of equations for the gap and for the magnetization determines the phase where the superconductivity and the ferromagnetism coexist. The system

can be written in terms of Bogoliubov excitations, which have the following dispersions relations:

$$\begin{aligned} E_1(\vec{k}) &= -\frac{JM}{2} - \sqrt{\epsilon^2(\vec{k}) + |\Delta(\vec{k})|^2} \\ E_2(\vec{k}) &= \frac{JM}{2} - \sqrt{\epsilon^2(\vec{k}) + |\Delta(\vec{k})|^2} \end{aligned} \quad (4)$$

where $\Delta(\vec{k})$ is the gap (1), J is the spin exchange constant, and $\epsilon(\vec{k}) = \frac{\vec{k}^2}{2m} - \mu$.

At zero temperature the equations take the form

$$M = \frac{1}{8\pi^2} \int_0^{\infty} dk k^2 \int_{-1}^1 dt [1 - \Theta(-E_2(k, t))] \quad (5)$$

$$\Delta = \frac{J^2 V_1}{32\pi^2} \int_{p_f - \Lambda}^{p_f + \Lambda} dk k^2 \int_{-1}^1 dt t^2 \frac{\Theta(-E_2(k, t))}{\sqrt{\epsilon^2(k) + \frac{3}{4\pi} t^2 \Delta^2}} \Delta \quad (6)$$

where $t = \cos \theta$.

The solution of the system which satisfies $\sqrt{\frac{3}{\pi}} \Delta < JM$ is discussed in [12]. In the present paper one looks for a solution of the system which satisfies

$$\sqrt{\frac{3}{\pi}} \Delta > JM \quad (7)$$

One is primarily interested in determining at what magnetization a superconductivity exists. The inequality Eq.(7) shows that the gap can not be arbitrarily small when the magnetization is finite. Hence the system undergoes the quantum phase transition from ferromagnetism to f-superconductivity with a jump. Approaching the quantum critical point from the ferromagnetic side, one sets the gap equal to zero in the equation for the magnetization (5) and considers the gap equation (6) with magnetization as a parameter. It is more convenient to consider the free energy as a function of the gap for the different values of the parameter M . To this purpose I introduce the dimensionless "gap" x and the parameters s, λ and g

$$x = \sqrt{\frac{3}{\pi}} \frac{m}{p_f^2} \Delta, \quad s = \frac{m}{p_f^2} JM, \quad \lambda = \frac{\Lambda}{p_f}, \quad g = \frac{J^2 V_1 m p_f}{8\pi^2} \quad (8)$$

Then the free energy is a function of x and depends on the parameters s, λ and g .

$$\begin{aligned} F(x) &= \frac{6m^2}{\pi p_f^4} (\mathcal{F}(x) - \mathcal{F}(0)) = x^2 + g \int_{1-\lambda}^{1+\lambda} dq q^2 \int_{-1}^1 dt \times \\ &\left[\left(s - \sqrt{(q^2 - 1)^2 + t^2 x^2} \right) \Theta(\sqrt{(q^2 - 1)^2 + t^2 x^2} - s) - \right. \\ &\left. \left(s - \sqrt{(q^2 - 1)^2} \right) \Theta(\sqrt{(q^2 - 1)^2} - s) \right] \end{aligned} \quad (9)$$

The dimensionless free energy $F(x)$ is depicted in Fig.1 for $\lambda = 0.08, g = 20$ and three values of the parameter $s, s = 0.8, s = 0.69$ and $s_{cr} = 0.595$. As the graph

shows, for some values of the microscopic parameters λ and g , and decreasing the parameter s (the magnetization), the system passes through a first order quantum phase transition. The critical values s_{cr} and x_{cr} satisfy $\frac{x_{cr}}{s_{cr}} = \sqrt{\frac{3}{\pi} \frac{\Delta_{cr}}{JM_{cr}}} > 1$ in agreement with Eq.(7).

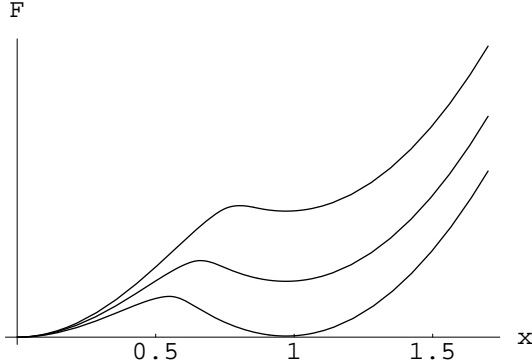


FIG. 1. The dimensionless free energy $F(x)$ as a function of dimensionless gap x . $\lambda = 0.08$, $g = 20$, $s_1 = 0.8$ (upper line), $s_2 = 0.69$ (middle line) and $s_{cr} = 0.595$ (lower line).

Varying the microscopic parameters beyond the critical values, one has to solve the system of equations (5,6). The equation of magnetization (5) shows that it is convenient to represent the gap in the form $\Delta = \sqrt{\frac{\pi}{3}} \kappa(M) JM$, where $\kappa(M) > 1$. Then the equation $E_2(k, t) = 0$, which defines the Fermi surface, has no solution if $-1 < t < -\frac{1}{\kappa(M)}$ and $\frac{1}{\kappa(M)} < t < 1$, and has two solutions

$$p_f^\pm = \sqrt{p_f^2 \pm m \sqrt{J^2 M^2 - \frac{3}{\pi} t^2 \Delta^2}} \quad (10)$$

when $-\frac{1}{\kappa(M)} < t < \frac{1}{\kappa(M)}$. The solutions (10) determine the two pieces of the Fermi surface. They stick together at $t = \pm \frac{1}{\kappa(M)}$, so that the Fermi surface is simple connected. The domain between pieces contributes to the magnetization M in Eq.(5), but it is cut out from the domain of integration in the gap equation Eq.(6).

When the magnetization approaches zero, one can approximate the equation for magnetization Eq.(5) substituting p_f^\pm from Eq.(10) in the difference $(p_f^+)^2 - (p_f^-)^2$ and setting $p_f^\pm = p_f$ elsewhere. Then, in this approximation, the magnetization is linear in Δ , namely

$$\Delta = \sqrt{\frac{\pi}{3}} J \kappa M \quad (11)$$

where $\kappa = \frac{mp_f J}{16\pi}$ is the small magnetization limit of $\kappa(M)$. The Eq.(11) is a solution if $mp_f J > 16\pi$ (see Eq.(7)). Substituting M from Eq.(11) in Eq.(6), one arrives at an equation for the gap. This equation can be solved in a standard way and the solution is

$$\Delta = \sqrt{\frac{16\pi}{3}} \frac{p_f \Lambda}{m} \exp \left[-\frac{24\pi^2}{mp_f J^2 V_1} - \frac{\pi}{4\kappa^3} + \frac{1}{3} \right] \quad (12)$$

Eqs (11,12) are the solution of the system Eqs.(5,6) near the quantum transition to paramagnetism. The second derivative of the free energy Eq.(9) with respect to the gap is positive when $\frac{mp_f J}{16\pi} > (\frac{21\pi}{16})^{\frac{1}{3}}$, hence the state where the superconductivity and the ferromagnetism coexist is stable.

When superconductivity and ferromagnetism coexist, the momentum distribution functions $n^\uparrow(p, t)$ and $n^\downarrow(p, t)$ of the spin-up and spin-down quasiparticles have complicated Fermi surfaces. One can write them in terms of the distribution functions of the Bogoliubov fermions

$$\begin{aligned} n^\uparrow(p, t) &= u^2(p, t) n_1(p, t) + v^2(p, t) n_2(p, t) \\ n^\downarrow(p, t) &= u^2(p, t) (1 - n_1(p, t)) + v^2(p, t) (1 - n_2(p, t)) \end{aligned} \quad (13)$$

where $u(p, t)$ and $v(p, t)$ are the coefficients in the Bogoliubov transformation. At zero temperature $n_1(p, t) = 1$, $n_2(p, t) = \Theta(-E_2(p, t))$, and the Fermi surface Eq.(10) manifests itself both in the spin up and spin-down momentum distribution functions. The functions are depicted in Fig.2 and Fig.3.

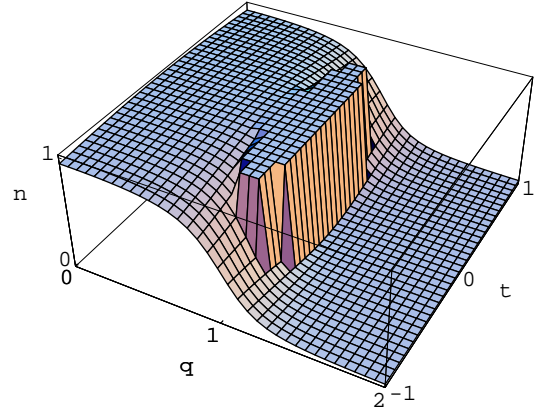


FIG. 2. The zero temperature momentum distribution n , for spin up fermions, as a function of $q = \frac{p}{p_f}$ and $t = \cos \theta$.

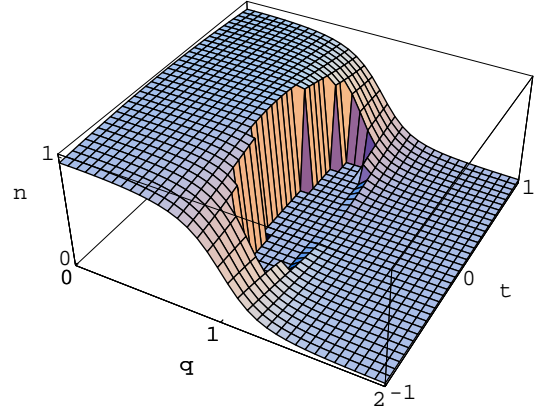


FIG. 3. The zero temperature momentum distribution n , for spin down fermions, as a function of $q = \frac{p}{p_f}$ and $t = \cos \theta$.

The existence of the Fermi surface explains the linear dependence of the specific heat at low temperature:

$$\frac{C}{T} = \frac{2\pi^2}{3} N(0) \quad (14)$$

Here $N(0)$ is the density of states on the Fermi surface. One can rewrite the $\gamma = \frac{C}{T}$ constant in terms of Elliptic Integral of the second kind $E(\alpha, x)$

$$\gamma = \frac{mp_f}{3\kappa(M)} \left[(1+s)^{\frac{1}{2}} E\left(\frac{\pi}{4}, \frac{2s}{s+1}\right) + (1-s)^{\frac{1}{2}} E\left(\frac{\pi}{4}, \frac{2s}{s-1}\right) \right]. \quad (15)$$

where $s < 1$ (see Eq.(8)). Eq.(15) shows that for $\kappa(M) \gg 1$ the specific heat constant γ is small in f-superconducting phase, which closely matches the experiments with $ZrZn_2$ and $URhGe$.

An important experimental fact is that $ZrZn_2$ and $URhGe$ are superconductors at ambient pressure as opposed to the existence of a quantum phase transition in UGe_2 . To comprehend this difference one considers the potential (2). The quantum phase transition results from the existence of a momentum cutoff Λ , above which the potential is repulsive. In turn, the cutoff existence follows from the relation $\beta = \frac{\rho}{2Mb} > 1$, which is true when the spin-wave approximation expression for the spin stiffness constant $\rho = M\rho_0$ is used. The spin wave approximation correctly describes systems with a large magnetization, for example UGe_2 . But in order to study systems with small magnetization, one has to account for the magnon-magnon interaction which changes the small magnetization asymptotic of ρ , $\rho = M^{1+\alpha}\rho_0$, where $\alpha > 0$. Then for a small M $\beta < 1$, and the potential is attractive for all momenta. Hence for systems which, at ambient pressure, are close to quantum critical point, as $ZrZn_2$ and $URhGe$, the magnon self-interaction renormalizes the spin fluctuations parameters so that the magnons dominate the pair formation and quantum phase transition can not be observed. But if one applies an external magnetic field, the magnon opens a gap proportional to the magnetic field. Increasing the magnetic field the paramagnon domination leads to first order quantum phase transition.

The proposed model of ferromagnetic superconductivity differs from the models discussed in [6–10] in many aspects. First, the superconductivity is due to the exchange of magnons, and the model describes in an unified way the superconductivity in UGe_2 , $ZrZn_2$ and $URhGe$. Second, the solution Eq.(11) shows that magnetization and superconductivity disappear simultaneously. It results from the equation of magnetization, which in turn is added to ensure that the fermions which form Cooper pairs are the same as those responsible for spontaneous magnetization. Hence, the fundamental assumption that superconductivity and ferromagnetism are caused by the same electrons leads to the experimentally observable

fact that the quantum phase transition is a transition to paramagnetic phase without superconductivity. Third, the paramagnons have pair-breaking effect. So, the understanding the mechanism of paramagnon suppression is crucial in the search for the ferromagnetic superconductivity with higher critical temperature. For example, one can build such a bilayer compound that the spins in the two layers are oriented in two non-collinear directions, and the net ferromagnetic moment is nonzero. The paramagnon in this phase is totally suppressed and the low lying excitations consist of magnons and additional spin wave modes with linear dispersion $\epsilon(k) \sim k$ [13]. If the new spin-waves are pair breaking, their effect is weaker than those of the paramagnons, and hence the superconducting critical temperature should be higher.

* Electronic address: naoum@phys.uni-sofia.bg

- [1] S. Saxena, P. Agarwal, K. Ahilan, F. M. Grosche, R. Haselwimmer, M. Steiner, E. Pugh, I. Walker, S. Julian, P. Monthoux, G. Lonzarich, A. Huxley, I. Sheikin, D. Braithwaite, and J. Flouquet, Nature (London) **406**, 587 (2000).
- [2] C. Pfleiderer, M. Uhlarz, S. Hayden, R. Vollmer, H.v. Löhneysen, N. Bernhoeft, and G. Lonzarich, Nature (London) **412**, 58 (2001).
- [3] D. Aoki, A. Huxley, E. Ressouche, D. Braithwaite, J. Flouquet, J-P. Brison, E. Lhotel, and C. Paulsen, Nature (London) **413**, 613 (2001).
- [4] A. Huxley, I. Sheikin, E. Ressouche, N. Kernavans, D. Braithwaite, R. Calemczuk, and J. Flouquet, Phys. Rev. B **63**, 144519 (2001).
- [5] N. Tateiva, T. Kobayashi, K. Hanazono, K. Amaya, Y. Haga, R. Settai, and Y. Onuki, J. Phys. Condens. Matter **13**, L17 (2001).
- [6] C. P. Enz, and P. T. Matthias, Science **201**, 828 (1978).
- [7] D. Fay, and J. Apple, Pys.Rev. B **22**, 3173 (1980).
- [8] T. Kirkpatrick, D. Belitz, T. Vojta and R. Narayanan Phys. Rev. Lett. **87**, 127003 (2001).
- [9] M. B. Walker, and K. V. Samokhin, Phys. Rev. Lett. **88**, 207001 (2002).
- [10] K. Machida, and T. Ohmi, Phys. Rev. Lett. **86**, 850 (2001).
- [11] N. F. Berk, and J. R Schrieffer, Phys. Rev. Lett. **17**, 433 (1966).
- [12] N. Karchev, cond-mat/0206534 (2002)
- [13] S. Sachdev, and T. Senthila, Ann.Phys.(NY) **251**, 76 (1996)